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BISTATIC SCATTERING CROSS SECTIONS OF A COMPOSITE ROUGH SURFACE

Saba Mudaliar

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APPROVED: Robert V. McGahan

ROBERT V. McGAHAN, Chief
Applied Electromagnetics Division
Electromagnetics & Reliability Directorate

FOR THE COMMANDER: John K Schindler

JOHN K. SCHINDLER
Director of Electromagnetics & Reliability

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The notion of shadowing is important in the analysis of scattering from rough surfaces using the Kirchhoff method. A key role is played here by the visibility function, which acts as a correction to the 'usual' scattering cross section. The surface under consideration is a normally distributed composite rough surface. Analytical results are provided for the bistatic scattering cross sections. Included herein is a derivation of the visibility function. Some properties of the visibility function are highlighted in a discussion. An important feature of this work is that our expressions for scattering cross sections, in contrast to those of Sancer (1969), are free from discontinuities and hence physically more appropriate.

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Contents

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Justification

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Dist	Avail and / or Special
A-1	

Illustrations

Bistatic Scattering Cross Sections of a Composite Rough Surface

1. INTRODUCTION

Perhaps the most commonly used method in the study of scattering from randomly rough surfaces is the Kirchhoff or physical optics method [Beckmann and Spizzichino, 1963; Bass and Fuks, 1979][1,2]. Inherent in this procedure is the notion of illuminated region and shadowed region. However it is assumed for the sake of simplicity that the entire surface is illuminated [Stogryn, 1967; Barrick, 1970][3,4]. Obviously this assumption is inappropriate when the angles of incidence and observation are large. Sancer [1969][5] has considered this problem and has derived 'shadow-corrected' scattering cross sections. Several others [Brown, 1978; Bahar, 1981][6,7] have used his method to account for shadowing. However we observe that his method leads to certain discontinuities in the bistatic scattering cross sections. This is clearly unphysical. Therefore, in this report an expression is derived for 'shadow-corrected' scattering cross section that does not suffer from the above-mentioned defect.

The contents of this report are organized as follows. The geometry of the problem is described in Section 2 while the statement of the problem is given in Section 3. In Section 4 the visibility function is derived. Section 5 contains expressions for 'shadow-corrected' bistatic scattering cross-

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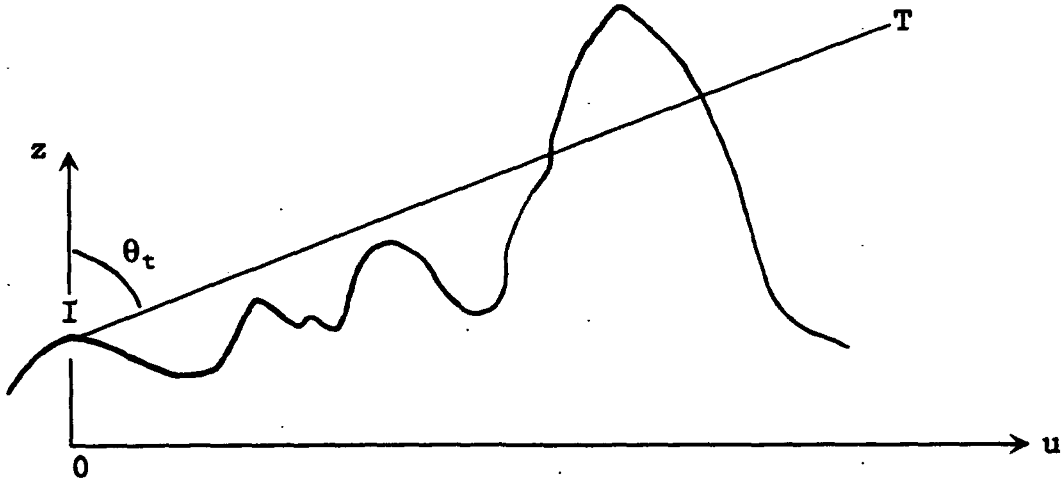
sections. In Section 6 we have a brief discussion of some characteristics of our results. The conclusions are stated in Section 7.

2. GEOMETRY OF THE PROBLEM

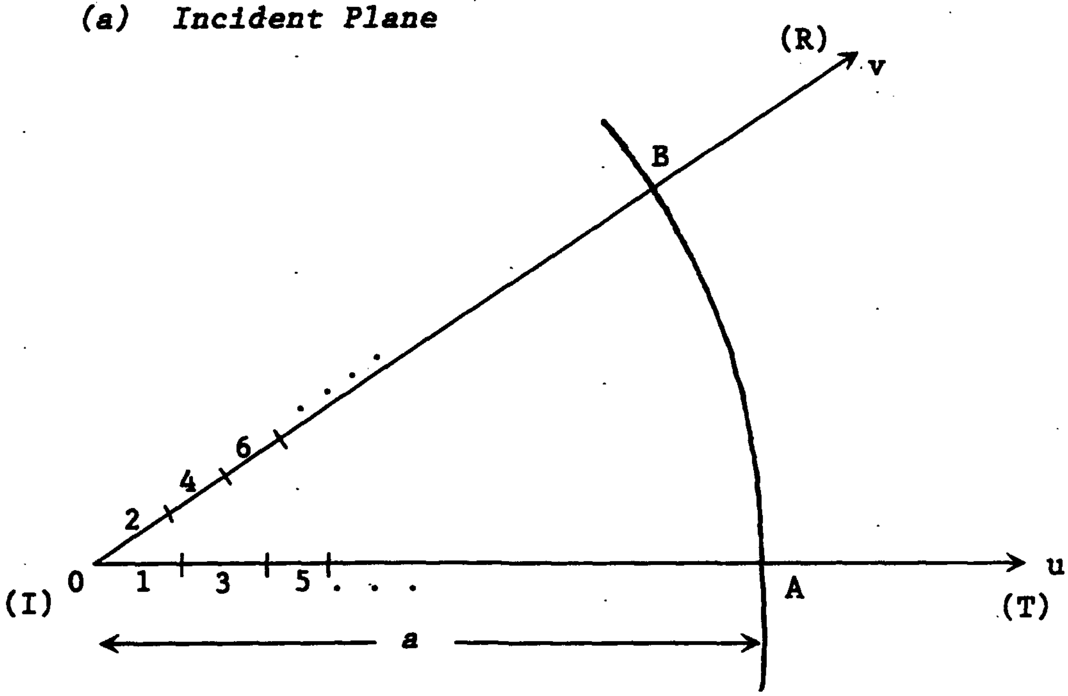
The rough surface $z = \xi(x,y)$ is represented here as a normally distributed stationary random process with zero mean and root mean square (rms) height h_1. The mean surface coincides with the xy-plane. Let T and R denote the locations of the transmitter and receiver and let θ_t and θ_r be respectively the elevation angles of IT and IR (see Figure 1). An alternative coordinate system (u,v) is chosen for the xy-plane such that the projection (onto the xy-plane) of IT coincides with the u-axis while the projection of IR coincides with the v-axis. The angle between the u- and v- axes is ϕ. The slopes of IT and IR are denoted as μ and ν; that is to say $\mu = \cot\theta_t$, $\nu = \cot\theta_r$. Let $p \equiv \frac{\partial\xi}{\partial u}$ and $q \equiv \frac{\partial\xi}{\partial v}$ denote the slopes of the surface at an arbitrary point $\xi(u,v)$. The corresponding rms slopes are denoted as s_p and s_q. The surface is assumed to be isotropic, which implies that $s_p = s_q \equiv s_1$. Further for a normally distributed surface we have $s_1 = \sqrt{2}\, h_1/\ell_1$ where ℓ is the correlation length of the surface.

3. STATEMENT OF THE PROBLEM

It is clear that when the angles of incidence and observation are large the phenomenon of shadowing must be incorporated in the Kirchhoff method. Sancer's shadowing function is intended to serve precisely this purpose. Our objective is to obtain 'shadow-corrected' scattering cross-sections appropriate for bistatic situations.



(a) Incident Plane



(b) Plan

Figure 1. Geometry of the Problem

The problem is to find the probability $V(\mu,\nu|\ \xi_0,\ p_0,\ q_0)$ that an arbitrary point $I(\xi_0,p_0,q_0)$ on the surface is visible to both transmitter and receiver. V is widely known as the 'shadowing function'. But it is apparent from its definition that the above label is a misnomer. A more appropriate name for V is 'visibility function'. We next take up the task of deriving V.

4. VISIBILITY FUNCTION

It is clear that for the point $I(\xi_0,p_0,q_0)$ to be visible to both transmitter and receiver the preliminary conditions $\mu > p_0$ and $\nu > q_0$, must be satisfied. Thus if the surface intercepts the rays IT and/or IR it must do so with slopes greater than that of the rays.

Consider the section of u- and v-axes enclosed in the circle of radius a about the origin. The line AOB is divided into 2n subsections such that the odd-numbered subsections are along the u-axis while the even-numbered subsections are along the v-axis (see Figure 1b).

We denote the event that the interception of one of the rays occurs within the subsection i as H_i. The complement of the event, namely, the event that neither of the rays is intercepted within the subsection i, is denoted as $\overline{H}_i$. Consider $V_a \equiv P(\overline{H}_1 \cap \overline{H}_2 \cap \ldots \cap \overline{H}_{2n})$. This denotes the probability that there is no interception of the rays by the surface within the radius a. V_a can be expanded as follows.

$$V_a = 1 - \sum_{i=1,3\,..} P(H_i) - \sum_{i=2,4\,..} P(H_i) + \sum_{\substack{i,j=1,3\,..\\ i\neq j}} 0.5\, P(H_i \cap H_j)$$

$$+ \sum_{\substack{i,j=2,4\,..\\ i\neq j}} 0.5\, P(H_i \cap H_j) + \sum_{i=1,3..} \sum_{j=2,4\,..} P(H_i \cap H_j) - \dots \tag{1}$$

In the limit as $n \to \infty$ and as the subsections become uniformly small we have

$$V_a = 1 - \int_0^a du_1\, g(u_1) - \int_0^a dv_1\, g(v_1) + .5 \int_0^a du_1 \int_0^a du_2\, g(u_1, u_2)$$

$$+ .5 \int_0^a dv_1 \int_0^a dv_2\, g(v_1, v_2) + \int_0^a du_1 \int_0^a dv_1\, g(u_1, v_1) - \dots \tag{2}$$

where the conditional density functions g's are derived in Appendix A.

Our visibility function V is now readily obtained as

$$V = \lim_{a \to \infty} V_a \tag{3}$$

But instead of using Eq. (2) we use an alternate representation that is convenient for our later approximations.

$$V_a = \exp\left\{ -\int_0^\infty du_1\, h(u_1) \;-\int_0^\infty dv_1\, h(v_1) + .5\int_0^\infty du_1 \int_0^\infty du_2\, h(u_1,u_2) \right.$$

$$\left. + .5\int_0^\infty dv_1 \int_0^\infty dv_2\, h(v_1,v_2) + \int_0^\infty du_1 \int_0^\infty dv_1\, h(u_1,v_1) - \ldots \right\} \tag{4}$$

where h's are the cumulant functions [Kubo, 1962][8]. All the cumulant functions can be expanded in terms of density functions. For example,

$$\begin{aligned} h(u_1) &= g(u_1) \\ h(v_1) &= g(v_1) \\ h(u_1,u_2) &= g(u_1,u_2) - g(u_1)\, g(u_2) \end{aligned} \tag{5}$$

We observe that when shadowing is significant the distances between crossing points become large. That is to say, $| u_i - u_j | > \ell_1$. In other words adjacent crossing points become uncorrelated. Further, an important property of the higher-order cumulant functions is that they vanish rapidly as the associated random quantities cend to become uncorrelated. Thus to a first-order, Eq. (4) may be approximated as

$$V = \exp\left\{ -\int_0^\infty du_1\, h(u_1) \;-\int_0^\infty dv_1\, h(v_1) + \int_0^\infty du_1 \int_0^\infty dv_1\, h(u_1,v_1) \right\} \tag{6}$$

From Eqs. (A3) and (5) we have

$$h(u_1) = \int_\mu^\infty dp\;(p-\mu)\; f(\xi_o+\mu u_1, p) \tag{7}$$

We further note that the slopes and height at any point on the surface are

uncorrelated. This implies that $f(\xi_o+\mu u_1,p) = f(\xi_o+\mu u_1)\, f(p)$. Thus on evaluating Eq. (7) we obtain

$$h(u_1) = f(\xi_o+\mu u_1)\ \Lambda_1(\mu) \tag{8a}$$

where

$$\Lambda_1(\mu) = \frac{s_1}{\sqrt{(2\pi)}} \exp\left[-\frac{\mu^2}{2s_1^2}\right] - \frac{\mu}{2}\ \mathrm{erfc}\left[\frac{\mu}{\sqrt{2}\ s_1}\right] \tag{8b}$$

Similarly we obtain $h(v_1)$ as

$$h(v_1) = f(\xi_o+\nu v_1)\ \Lambda_1(\nu) \tag{9}$$

We note from Eq. (5) that

$$h(u_1,v_1) = g(u_1,v_1) - g(u_1)\ g(v_1) \tag{10}$$

where $g(u_1,v_1)$ is defined in Appendix A as

$$g(u_1,v_1) = \int_\mu^\infty dp\ (p-\mu) \int_\nu^\infty dq\ (q-\nu)\ f(\xi_o+\mu u_1,p;\xi_o+\nu v_1,q) \tag{11}$$

In principle, Eq. (11) can be evaluated using the general expression for $f(\overline{X})$ as derived in Appendix B. But it appears too complicated to obtain analytical results. We therefore focus attention on two special cases.

<u>Case 1 : Large ϕ; $r/\ell_1 \gg 1$</u>

In this case $r/\ell_1 \gg 1$, and as shown in Appendix C, $f(\overline{X})$ assumes the simplified form

$$f(\overline{X}) = f(X_1)\ f(X_2)\ f(X_3)\ f(X_4) \tag{12}$$

Hence

$$g(u_1,v_1) = \int_\mu^\infty dp\ (p-\mu) \int_\nu^\infty dq\ (q-\nu)\ f(\xi_o+\mu u_1)$$

$$\cdot\ f(p)\ f(\xi_o+\nu v_1)\ f(q)$$

$$= f(\xi_o+\mu u_1)\ f(\xi_o+\nu v_1)\ \Lambda_1(\mu)\ \Lambda_1(\nu) \tag{13}$$

From Eqs. (13), (10), (8), and (9)

$$h(u_1,v_1) = 0 \tag{14}$$

Thus the visibility function in this case is

$$V = \exp\left\{ - \int_0^\infty du_1\ h(u_1) - \int_0^\infty dv_1\ h(v_1) \right\} \tag{15}$$

Using Eq. (8) we have

$$\int_0^\infty du_1\ h(u_1) = \int_0^\infty du_1\ f(\xi_o+\mu u_1)\ \Lambda_1(\mu)$$

$$= \frac{\Lambda_1(\mu)}{2\mu}\ \mathrm{erfc}\left[\frac{\xi_o}{\sqrt{2h_1}}\right] \tag{16}$$

Likewise

$$\int_0^\infty dv_1\ h(v_1) = \frac{\Lambda_1(\nu)}{2\nu}\ \mathrm{erfc}\left[\frac{\xi_o}{\sqrt{2h_1}}\right] \tag{17}$$

Thus

$$V = \exp\left[-\frac{1}{2}\operatorname{erfc}\left[\frac{\xi_o}{\sqrt{2}h_1}\right]\left\{\tilde{\Lambda}_1(\mu) + \tilde{\Lambda}_1(\nu)\right\}\right] \tag{18}$$

where

$$\tilde{\Lambda}_1(\mu) = \frac{\Lambda_1(\mu)}{\mu} \tag{19a}$$

$$\tilde{\Lambda}_1(\nu) = \frac{\Lambda_1(\nu)}{\nu} \tag{19b}$$

As explained earlier, V represents the conditional probability that a point on the surface with height ξ_o and slopes p_o, q_o is visible to both transmitter and receiver. We also mentioned that to avoid self-shadowing the inequalities $p_o < \mu$ and $q_o < \nu$ must hold. The above ideas are condensed in the following expression for V, or more specifically $V(\mu,\nu|\ \xi_o,p_o,q_o)$.

$$V(\mu,\nu|\ \xi_o,p_o,q_o) = \mathcal{H}(\mu-p_o)\ \mathcal{H}(\nu-q_o)$$

$$\cdot \exp\left[-\frac{1}{2}\operatorname{erfc}\left[\frac{\xi_o}{\sqrt{2}h_1}\right]\left\{\tilde{\Lambda}_1(\mu) + \tilde{\Lambda}_1(\nu)\right\}\right] \tag{20}$$

where $\mathcal{H}(t)$ is the Heaviside function with the usual definition that $\mathcal{H}(t) = 0$ for $t < 0$ and $\mathcal{H}(t) = 1$ for $t \geq 0$. From Eq. (20) we can obtain the visibility function $V(\mu,\nu|\ p_o,q_o)$ as

$$V(\mu,\nu|\ p_o,q_o) = \int_{-\infty}^{\infty} d\xi_o\ f(\xi_o)\ V(\mu,\nu|\ \xi_o,p_o,q_o) \tag{21}$$

Substituting Eq. (20) into Eq. (21) we have

$$V(\mu,\nu|\ p_o,q_o) = \mathcal{H}(\mu-p_o)\ \mathcal{H}(\nu-q_o)\ \frac{1}{\delta}\left[1 - e^{-\delta}\right] \tag{22}$$

where

$$\delta = \tilde{\Lambda}_1(\mu) + \tilde{\Lambda}_1(\nu) \tag{23}$$

Suppose that it is given that $p_o < \mu$ and $q_o < \nu$. The visibility function in this situation, denoted as $\bar{V}(\mu,\nu)$ is given as

$$\bar{V}(\mu,\nu) = \frac{1}{\delta}\left[1 - e^{-\delta}\right] \tag{24}$$

<u>Case 2: $r/\ell_1 = 0$; $\mu = \nu$</u>

In this case $f(\bar{X})$ takes the form (see Appendix C)

$$f(\bar{X}) = f(X_1)\ f(X_2)\ \delta(X_1-X_3)\ \delta(X_2-X_4)$$

so that

$$f(\xi_o+\mu u_1,p;\xi_o+\nu v_1,q) = f(\xi_o+\mu u_1)\ f(p)\ \delta(u_1-v_1)\ \delta(p-q) \tag{25}$$

Hence

$$g(u_1,v_1) = \int_\mu^\infty dp\ (p-\mu) \int_\nu^\infty dq\ (q-\nu)$$

$$\cdot\ f(\xi_o+\mu u_1)\ f(p)\ \delta(u_1-v_1)\ \delta(p-q)$$

$$= f(\xi_o+\mu u_1)\ \delta(u_1-v_1) \int_\mu^\infty dp\ (p-\mu)^2\ f(p)$$

$$= f(\xi_o+\mu u_1)\ \delta(u_1-v_1)\ \Lambda_2(\mu) \tag{26}$$

where

$$\Lambda_2(\mu) = \frac{\mu \sigma_1}{\surd(2\pi)} \exp\left[-\frac{\mu^2}{2\sigma_1^2}\right] \left\{ -1 + \surd(\pi/2)\ \sigma_1^3\ \operatorname{erf}\left[\frac{\mu}{\sqrt{2}\,\sigma_1}\right] \right\}$$

$$+\ 0.5\ \mu^2\ \operatorname{erfc}\left[\frac{\mu}{\sqrt{2}\,\sigma_1}\right] \tag{27}$$

It follows from Eqs. (10), (26), (16), and (17) that

$$\int_0^\infty du_1 \int_0^\infty dv_1\ h(u_1,v_1) =$$

$$= \int_0^\infty du_1 \int_0^\infty dv_1 \left[g(u_1,v_1) - g(u_1)\ g(v_1) \right]$$

$$= \int_0^\infty du_1 \int_0^\infty dv_1\ f(\xi_o+\mu u_1)\ \delta(u_1-v_1)\ \Lambda_2(\mu)$$

$$- \int_0^\infty du_1\ g(u_1) \int_0^\infty dv_1\ g(v_1)$$

$$= \frac{1}{2}\ \tilde{\Lambda}_2(\mu)\ \text{erfc}\left[\frac{\xi_o}{\sqrt{2}h_1}\right]$$

$$- \frac{1}{4}\ \tilde{\Lambda}_1^2(\mu)\ \text{erfc}^2\left[\frac{\xi_o}{\sqrt{2}h_1}\right] \tag{28}$$

where

$$\tilde{\Lambda}_2(\mu) = \frac{\Lambda_2(\mu)}{\mu} \tag{29}$$

From Eqs. (28), (16), (17), and (6) we have

$$V(\mu,\mu|\xi_o,p_o) = \exp\left\{ - \tilde{\Lambda}_1^2(\mu)\ \text{erfc}^2\left[\frac{\xi_o}{\sqrt{2}h_1}\right] - \left[\ 2\ \tilde{\Lambda}_1(\mu) - \tilde{\Lambda}_2(\mu)\ \right] \text{erfc}\left[\frac{\xi_o}{\sqrt{2}h_1}\right] \right\} \tag{30}$$

The visibility function independent of ξ_o is given as

$$V(\mu,\mu|p_o) = \int_{-\infty}^{\infty} d\xi_o \; f(\xi_o) \; V(\mu,\mu|\; \xi_o,p_o)$$

$$= \frac{\sqrt{\pi}}{4\alpha} \; \exp\left[\frac{\beta^2}{4\alpha^2} \right]$$

$$\cdot \left\{ \operatorname{erf}\left[\frac{\beta}{2\alpha} \right] - \operatorname{erf}\left[2\alpha + \frac{\beta}{2\alpha} \right] \right\} \tag{31}$$

where

$$\alpha = \tilde{\Lambda}_1(\mu) \tag{32a}$$

$$\beta = 2\tilde{\Lambda}_1(\mu) - \tilde{\Lambda}_2(\mu) \tag{32b}$$

As explained in case 1 the above expression assumes that $\mu > p_o$ and hence it is identical to $\tilde{V}(\mu,\mu|p_o)$.

5. SCATTERING CROSS SECTIONS

Analytic solutions for scattering cross sections are available at present only for certain types of random surfaces. The one among them of particular interest to us here is the composite rough surface [Beckmann, 1965; Bahar, 1981][9,7] . Both for its generality and its utility in various applications, the composite rough surface deserves attention. As the name suggests, this is composed of a small scale rough surface superposed on a large scale rough surface. We characterize the large scale roughness by rms height h_1 and correlation length ℓ_1. The corresponding parameters for small scale roughness are h_2 and ℓ_2. In practical applications where the composite

rough surface model is useful it is found that the two scales are uncorrelated. This is primarily because the large scale roughness and the small scale roughness are usually generated by different physical mechanisms. In this case it has been shown [Semenov, 1966; Barrick and Peake, 1968][10,11] that the problem can be broken up into two parts. Scattering due to large scale roughness is determined using the Kirchhoff method [Beckmann and Spizzichino, 1963][1] while scattering due to small scale roughness is evaluated using the perturbation method [Rice, 1951][12]. The above two quantities are then added incoherently to obtain the scattering due to the composite rough surface.

It should be mentioned that the above procedure is valid only as a single scattering approximation. Thus we agree to ignore multiple scattering effects. Still there is another factor to be taken into consideration - the phenomenon of shadowing. Sancer [1969][5] has shown that for large scale rough surfaces the 'shadow-correction' is incorporated by multiplying the usual scattering cross sections by the visibility function $\tilde{V}(\mu,\nu)$. Further, Brown [1978][6] has formally derived the scattering cross sections of a composite rough surface by a perturbation scheme to deal with the small-scale fluctuations of the underlying large scale rough surface. This leads to the conclusion that the 'shadow-correction' to the composite rough surface may be obtained similarly by using the same visibility function. In other words, the shadow-corrected scattering cross section of the composite rough surface is written as

$$\gamma_{cd} = \tilde{V}(\mu,\nu) \left[\gamma_{cd}^{(s)} + \gamma_{cd}^{(\ell)} \right], \qquad \left\{ c,d \right\} = \left\{ V,H \right\} \tag{33}$$

where $\tilde{V}(\mu,\nu)$ is the visibility function derived in Section 4. The scattering cross sections, $\gamma_{cd}^{(s)}$ and $\gamma_{cd}^{(\ell)}$, are due, respectively, to the small and large scale rough surfaces that constitute the given composite rough surface. Barrick and Peake [1967][13] have calculated $\gamma_{cd}^{(s)}$) and . $\gamma_{cd}^{(\ell)}$. Their results are quoted here.

(i) $$\gamma_{cd}^{(s)} = \frac{4}{\pi} k_o^4 h_2^2 \cos^2\theta_i \cos^2\theta_s \, |A_{cd}|^2 \, J^{(s)} \tag{34}$$

where

$$J^{(s)} = \pi \ell_2^2 \exp\left[-\frac{1}{4} k_o^2 \ell_2^2 (\eta_x^2 + \eta_y^2) \right] \tag{35}$$

$$\eta_x = \sin\theta_i - \sin\theta_s \cos\phi \tag{36a}$$

$$\eta_y = \sin\theta_s \sin\phi \tag{36b}$$

$$A_{HH} = \frac{\left[\mu_r^2(\epsilon_r-1)\cos\phi + (\mu_r-1)(\mu_r\sin\theta_i\sin\theta_s - t_it_s\cos\phi)\right]}{(t_i+\mu_r\cos\theta_i)\,(t_s+\mu_r\cos\theta_s)} \tag{37a}$$

$$A_{VH} = \frac{t_i\epsilon_r(\mu_r-1) - t_s\mu_r(\epsilon_r-1)}{(t_i+\mu_r\cos\theta_i)(t_s+\epsilon_r\cos\theta_s)} \sin\phi \tag{37b}$$

$$A_{HV} = \frac{t_s\mu_r(\epsilon_r-1) - t_i\epsilon_r(\mu_r-1)}{(t_s+\mu_r\cos\theta_s)(t_i+\epsilon_r\cos\theta_i)} \sin\phi \tag{37c}$$

$$A_{VV} = \frac{\left[\epsilon_r^2(\mu_r-1)\cos\phi + (\epsilon_r-1)(\epsilon_r\sin\theta_i\sin\theta_s - t_i t_s\cos\phi)\right]}{(t_i+ \epsilon_r\cos\theta_i)\ (t_s+ \epsilon_r\cos\theta_s)} \tag{37d}$$

$$t_\chi = \left[\epsilon_r\mu_r - \sin^2\theta_\chi \right]^{1/2} , \ \chi = i,s \tag{38}$$

(ii)

$$\gamma_{cd}^{(\ell)} = | B_{cd}|^2 J^{(\ell)} \tag{39}$$

where

$$J^{(\ell)} = \frac{4}{s_1^2\eta_z^2} \exp\left[- \frac{1}{s_1^2} \left\{ \frac{\eta_x^2 + \eta_y^2}{\eta_z^2} \right\} \right] \tag{40}$$

$$\eta_z = - \cos\theta_i - \cos\phi \tag{41}$$

$$B_{HH} = - \frac{1}{a_1a_4} \left[R_H\ a_2a_3 + R_V \sin\theta_i \sin\theta_s \sin^2\phi\right] \tag{42a}$$

$$B_{VH} = \frac{\sin\phi}{a_1a_4} \left[- R_H\ a_3 \sin\theta_i + R_V\ a_2 \sin\theta_s \right] \tag{42b}$$

$$B_{HV} = \frac{\sin\phi}{a_1a_4} \left[R_H\ a_2 \sin\theta_s - R_V\ a_3 \sin\theta_i \right] \tag{42c}$$

$$B_{VV} = - \frac{1}{a_1a_4} \left[R_H \sin\theta_i \sin\theta_s \sin^2\phi + R_V\ a_2a_3 \right] \tag{42d}$$

where R_H and R_V are the Fresnel reflection coefficients with angle of incidence ψ (defined below) and

$$a_1 = 1 + \sin\theta_i \sin\theta_s \cos\phi - \cos\theta_i \cos\theta_s \tag{43a}$$

$$a_2 = \cos\theta_i \sin\theta_s + \sin\theta_i \cos\theta_s \cos\phi \tag{43b}$$

$$a_3 = \sin\theta_i \cos\theta_s + \cos\theta_i \sin\theta_s \cos\phi \tag{43c}$$

$$a_4 = \cos\theta_i + \cos\theta_s \tag{43d}$$

$$\psi = \arccos\left\{.5\left[1 - \sin\theta_i \cos\theta_s \cos\phi + \cos\theta_i \cos\theta_s\right]\right\}^{.5} \tag{44}$$

The important symbols in the above expressions are defined next. The elevation angles of the incident and the scattered rays are θ_i and θ_s. They are identical to θ_t and θ_r mentioned in the previous sections of this report. The propagation constant in free space, the condition above the rough surface, is k_o. The medium below the rough surface is of relative permittivity ε_r and relative permeability μ_r. The Fresnel reflection coefficients for horizontal polarization R_H and for vertical polarization R_V are given as

$$R_H = \frac{\mu_r \cos\psi - (\varepsilon_r\mu_r - \sin^2\psi)^{.5}}{\mu_r \cos\psi + (\varepsilon_r\mu_r - \sin^2\psi)^{.5}} \tag{45a}$$

$$R_V = \frac{\varepsilon_r \cos\psi - (\varepsilon_r\mu_r - \sin^2\psi)^{.5}}{\varepsilon_r \cos\psi + (\varepsilon_r\mu_r - \sin^2\psi)^{.5}} \tag{45b}$$

6. DISCUSSION

Our interest here is to study the expressions for the shadow-corrected bistatic scattering cross-sections given in Eq. (36). Barrick and Peake [1967][13] have elaborately analyzed the characteristics of $\gamma_{cd}^{(s)}$ and $\gamma_{cd}^{(\ell)}$; so it suffices for us here to study the behavior of the visibility function $\tilde{V}(\mu,\nu)$. We notice that $\tilde{V}(\mu,\nu)$ has a rather complicated functional dependence on the physical parameters μ, ν, θ_i, θ_r and ϕ. Therefore, we must be content with looking at two limiting cases.

As the visibility parameters $\tilde{\mu} \equiv \frac{\mu}{\sqrt{2}\,s_1}$ and $\tilde{\nu} \equiv \frac{\nu}{\sqrt{2}\,s_1}$ become small it is clear that the surface area of visibility diminishes, which implies that $\tilde{V}(\mu,\nu)$ should vanish in the above limit. In order to verify this physical fact we proceed as follows. On using the asymptotic expression $\mathrm{erfc}(t) \simeq 1 - 2t/\sqrt{\pi}$ for $t \ll 1$ we obtain from Eq. (23) the result,

$$\tilde{V}(\mu,\nu) \simeq 2\sqrt{\pi}\,\frac{\overline{\tilde{\mu}\tilde{\nu}}}{\tilde{\mu} + \tilde{\nu}} \quad ; \quad \tilde{\mu}\ ,\ \tilde{\nu} \ll 1 \tag{46}$$

We thus see that $\tilde{V}(\mu,\nu)$ linearly approaches zero as $\tilde{\mu}, \tilde{\nu} \to 0$.

On the other hand $\tilde{\mu}$, $\tilde{\nu} \gg 1$ when we use the asymptotic expression $\mathrm{erfc}(t) \simeq t^{-1}\exp(-t^2)/\sqrt{\pi}$ for $t \gg 1$ and find that $\tilde{V}(\mu,\nu)$ approaches unity. This is an intuitively satisfying physical result. For in this limit virtually the entire surface is visible and hence there is no need for 'shadow- correction'.

It is only in this region that the results of Barrick and Peake [1967][13] are strictly valid. Our results are hence the natural generalizations of theirs.

It is apparent that our shadow-corrected bistatic scattering cross-sections have several other interesting characteristics. But since their analytic forms are very complicated it seems that they can be further studied only by considering some numerical examples.

7. CONCLUSION

We have considered a normally distributed composite rough surface and derived the shadow-corrected scattering cross sections. This contrasts to several other works where the notion of shadowing is ignored. A key role is played in the analysis by the visibility function. Physically this function represents the fraction of the surface visible to both the transmitter and the receiver. A detailed derivation of the visibility function is provided. Also included is a discussion of the properties of the visibility function. This work may be regarded as an improvement of the work of Sancer [1969][5] whose expressions for scattering cross sections contain certain erroneous discontinuities. This work also has important practical applications. Several radars presently must operate at grazing angles. Shadowing is a dominating physical phenomenon in such situations. Since many modern radars operate in the bistatic mode and since the composite rough surface is a very useful practical model we anticipate that our studies in this report will serve to enhance the performance capabilities of such radars.

Appendix A

Conditional Density Functions

According to its definition, $g(u_1)\Delta u_1$ represents the conditional probability that the surface intercepts the ray IT in the interval $(u_1, u_1+\Delta u_1)$ with $p > \mu$ given that the height at I is ξ_o and its slope is p_o. Suppose the surface crosses IT at u in $(u_1, u_1+\Delta u_1)$. Then at the crossing point, $\xi_o+\mu u = \xi(u_1)+p(u-u_1)$. This implies that $\xi_o+\mu u_1 > \xi(u_1) > \xi_o+\mu u_1 - (p-\mu)\Delta u_1$. Let the event that the surface crosses IT with $p > \mu$ in the interval $(u_1, u_1+\Delta u_1)$ be $\mathcal{A}$. Then

$$P(\mathcal{A}) = \int_{\mu}^{\infty} dp \int_{\xi_o+\mu_1 u_1-(p-\mu)\Delta u_1}^{\xi_o+\mu u_1} d\xi \; f(\xi,p)$$

$$\approx \Delta u_1 \int_{\mu}^{\infty} dp \; (p-\mu) \; f(\xi_o+\mu u_1, p) \tag{A1}$$

where the approximate result is obtained using the mean value theorem. Throughout this report we denote normal density functions by f's. For example $f(\xi,p)$ denotes the joint normal density function of slope and height at u.

Now if the point I from which the ray emanates is of height ξ_o and slope p_o we can deduce the conditional probability $g_1(u_1)\Delta u_1$ from Eq. (A1) as follows.

$$g(u_1)\Delta u_1 = \frac{\Delta u_1}{f(\xi_o,p_o)} \int_{\mu}^{\infty} dp \; (p-\mu) \; f(\xi_o+\mu u_1, \; p;\xi_o,p_o) \tag{A2}$$

When shadowing is significant it is apparent that the distances between adjacent crossing points are large. This means that adjacent crossing points are weakly correlated. Then we can approximate $f(\xi_o+\mu u_1, p;\xi_o,p_o)$ as $f(\xi_o+\mu u_1,p)\, f(\xi_o,p_o)$. Thus we have from Eq. (A2)

$$g(u_1) = \int_{\mu}^{\infty} dp\ (p-\mu)\ f(\xi_o+\mu u_1,\ p) \tag{A3}$$

Similarly we can proceed to obtain other conditional density functions. For example,

$$g(v_1) = \int_{\nu}^{\infty} dq\ (q-\nu)\ f(\xi_o+\nu v_1,\ q) \tag{A4}$$

$$g(u_1,v_1) = \int_{\mu}^{\infty} dp\ (p-\mu) \int_{\nu}^{\infty} dq\ (q-\nu)\ f(\xi_o+\mu u_1,p;\xi_o+\nu v_1,q) \tag{A5}$$

Appendix B

The Density Function f $(\xi(u,0), p; \xi(0,v), q)$

Let

$$X_1 = \xi(u,0) \tag{B1a}$$

$$X_2 = \frac{\partial}{\partial u}\,\xi(u,0) = p \tag{B1b}$$

$$X_3 = \xi(0,v) \tag{B1c}$$

$$X_4 = \frac{\partial}{\partial v}\,\xi(0,v) = q \tag{B1d}$$

Now our object in this appendix is to compute the fourth-order normal density function $f(X_1, X_2, X_3, X_4)$ or $f(\bar{X})$. In terms of its characteristic function we have

$$f(\bar{X}) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} d^4\bar{W}\;\; \Phi(\bar{W})\;\; \exp[-\, i\bar{X}\cdot\bar{W}\,] \tag{B2}$$

Noting that X_1, X_2, X_3, X_4 have zero means we have

$$\Phi(\bar{W}) = \exp\;[-\; i\bar{W}\cdot\bar{\bar{K}}\cdot\bar{W}\;] \tag{B3}$$

where $\bar{\bar{K}}$ is the covariance matrix defined as

$$K_{ij} = \mathrm{cov}\;(X_i,\; X_j) = \langle X_i X_j \rangle \tag{B4}$$

Since the surface is assumed to be statistically homogeneous and isotropic the height-height correlation function $C(\bar{r})$ is given as

$$C(r) = C(|r_1 - r_2|) = \langle \xi(\bar{r}_1 - \bar{r}_2) \rangle \tag{B5}$$

In terms of $C(\bar{r})$ the covariance matrix $\bar{\bar{K}}$ takes the form,

$$\bar{\bar{K}} = \begin{matrix} C(0) & 0 & C(r) & C_v(r) \\ 0 & -C''(0) & C_u(r) & C_{uv}(r) \\ C(r) & C_u(r) & C(0) & 0 \\ C_v(r) & C_{uv}(r) & 0 & -C''(0) \end{matrix} \tag{B6}$$

where

$$C''(r) = \frac{d^2C(r)}{dr^2}$$

$$C_u(r) = \frac{\partial C(r)}{\partial u}$$

$$C_v(r) = \frac{\partial C(r)}{\partial v}$$

$$C_{uv}(r) = \frac{\partial^2 C(r)}{\partial u \, \partial v} \tag{B7}$$

$$r^2 = u^2 + v^2 - 2uv \cos\phi \tag{B8}$$

For definiteness and for the purpose of discussion we assume that the surface heights have the following correlation function

$$C(r) = \mathscr{h}_1^2 \exp \left(- \frac{r^2}{\ell_1^2} \right) \tag{B9}$$

In this case the various elements of the covariance matrix are given as follows.

$$C(0) = \mathscr{h}_1^2 \tag{B10}$$

$$C''(0) = - \mathscr{s}_1^2 \tag{B11}$$

$$C_u(r) = - \mathscr{s}_1^2 \, (u\text{-}v \cos\phi) \exp \left(- (r/\ell_1)^2 \right) \tag{B12}$$

$$C_v(r) = - \mathscr{s}_1^2 \, (v\text{-}u \cos\phi) \exp \left(- (r/\ell_1)^2 \right) \tag{B13}$$

$$C_{uv}(r) = - \mathscr{s}_1^2 \exp \left(- (r/\ell_1)^2 \right)$$

$$\cdot \left\{ \cos\phi + \frac{2}{\ell_1^2} (u\text{-}v \cos\phi) \, (v\text{-}u \cos\phi) \right\} \tag{B14}$$

Appendix C

Density Function For Two Special Cases

<u>Case 1 : large ϕ, $r/\ell_1 \gg 1$</u>

From Eqs. (B9)-(B14) we note that

$$C(r) = C_u(r) = C_v(r) = C_{uv}(r) \simeq 0 \tag{C1}$$

Thus

$$\bar{\bar{K}} = \begin{array}{cccc} r_1^2 & 0 & 0 & 0 \\ 0 & s_1^2 & 0 & 0 \\ 0 & 0 & r_1^2 & 0 \\ 0 & 0 & 0 & s_1^2 \end{array} \tag{C2}$$

The characteristic function is then

$$\Phi(\bar{W}) = \exp\left\{ -\frac{1}{2} \left(r_1{}^2 W_1{}^2 + s_1{}^2 W_2{}^2 + r_1{}^2 W_3{}^2 + s_1{}^2 W_4{}^2\right) \right\} \tag{C3}$$

Substituting Eq. (C3) into Eq. (B2) we have

$$f(\bar{X}) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} d^4\bar{W} \exp\left\{ -\frac{1}{2} \left(r_1{}^2 W_1{}^2 + s_1{}^2 W_2{}^2 + r_1{}^2 W_3{}^2 + s_1{}^2 W_4{}^2\right) \right\}$$

$$\cdot \exp\left[-i \left(X_1W_1 + X_2W_2 + X_3W_3 + X_4W_4\right)\right] \tag{C4}$$

$$= I_1 \; I_2 \; I_3 \; I_4$$

where

$$I_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dW_1 \exp\left[- .5\ \sigma_1^2\ W_1^2 - i\ X_1 W_1 \right] \tag{C5}$$

$$I_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dW_2 \exp\left[- .5\ \sigma_1^2\ W_2^2 - i\ X_2 W_2 \right] \tag{C6}$$

$$I_3 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dW_3 \exp\left[- .5\ \sigma_1^2\ W_3^2 - i\ X_3 W_3 \right] \tag{C7}$$

$$I_4 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dW_4 \exp\left[- .5\ \sigma_1^2\ W_4^2 - i\ X_4 W_4 \right] \tag{C8}$$

These integrals are elementary and the results are

$$I_1 = \frac{1}{\sqrt{(2\pi)}\ \sigma_1} \exp\left[- \frac{X_1^2}{2\sigma_1^2} \right] = f(X_1) \tag{C9a}$$

$$I_2 = f(X_2) \tag{C9b}$$

$$I_3 = f(X_3) \tag{C9c}$$

$$I_4 = f(X_4) \tag{C9d}$$

Thus

$$f(\bar{X}) = f(X_1)\, f(X_2)\, f(X_3)\, f(X_4) \tag{C10}$$

Case 2: $\phi = 0$; $r = 0$; $\mu = \nu$

In this case

$$C(0) = h_1^2$$

$$C_u(0) = C_v(0) = 0$$

$$C_{uv}(0) = s_1^2$$

Thus the covariance matrix takes the following form

$$\bar{\bar{K}} = \begin{pmatrix} h_1^2 & 0 & h_1^2 & 0 \\ 0 & s_1^2 & 0 & s_1^2 \\ h_1^2 & 0 & h_1^2 & 0 \\ 0 & s_1^2 & 0 & s_1^2 \end{pmatrix} \tag{C11}$$

The four eigenvalues of $\bar{\bar{K}}$ are $2h_1^2$, $2s_1^2$, 0, 0 . Corresponding to them we may choose four linearly independent eigenvectors as

$$\bar{\eta}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \bar{\eta}^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \bar{\eta}^{(3)} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \quad \bar{\eta}^{(4)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

Let

$$\bar{\bar{P}} = \frac{1}{\sqrt{2}} \left[\bar{\eta}^{(1)}, \bar{\eta}^{(2)}, \bar{\eta}^{(3)}, \bar{\eta}^{(4)} \right] \tag{C12}$$

Note that we have normalized the eigenvectors. By a well known theorem in matrix analysis,

$$\bar{\bar{P}}^{-1} \bar{\bar{K}} \bar{\bar{P}} = \bar{\bar{\Lambda}} \tag{C13}$$

where

$$\bar{\bar{\Lambda}} = \mathrm{diag} \left(2h_1^2 , 2s_1^2 , 0 , 0 \right). \tag{C14}$$

Also we know that normalized eigenvectors form an orthonormal set. This implies

$$\bar{\bar{P}} \bar{\bar{P}}^t = \bar{\bar{P}}^t \bar{\bar{P}} = \bar{\bar{I}} \tag{C15}$$

But since in our case $\bar{\bar{P}} = \bar{\bar{P}}^t$ we have

$$\bar{\bar{P}} = \bar{\bar{P}}^{-1} \tag{C16}$$

The density function $f(\bar{X})$ is given as

$$f(\bar{X}) = (2\pi)^{-4} \int_{-\infty}^{\infty} d^4\bar{W} \ \exp \left[- 0.5 \ \bar{W} \cdot \bar{\bar{K}} \cdot \bar{W} - i\bar{X} \cdot \bar{W} \right] \tag{C17}$$

On making the transformation

$$\bar{W} = \bar{\bar{P}} \cdot \bar{Z} \tag{C18}$$

$$d^4\bar{W} = J\, d^4\bar{Z} \tag{C19}$$

where J is the Jacobian of the transformation. In our case

$$J = \det \bar{\bar{P}} = 1 \tag{C20}$$

Thus

$$f(\bar{X}) = (2\pi)^{-4} \int_{-\infty}^{\infty} d^4\bar{Z} \exp\left[- 0.5\ \bar{Z}\cdot\bar{\bar{P}}\ \bar{\bar{K}}\ \bar{\bar{P}}\cdot\bar{Z} - i\bar{X}\cdot\bar{\bar{P}}\cdot\bar{Z} \right]$$

$$= (2\pi)^{-4} \int_{-\infty}^{\infty} d^4\bar{Z} \exp\left[- 0.5\ \bar{Z}\cdot\bar{\bar{\Lambda}}\cdot\bar{Z} - i\bar{X}\cdot\bar{\bar{P}}\cdot\bar{Z} \right] \tag{C21}$$

But

$$0.5\ \bar{Z}\cdot\bar{\bar{\Lambda}}\cdot\bar{Z} = \lambda_1^2\, Z_1^2 + \delta_1^2\, Z_2^2 \tag{C22}$$

$$\bar{X}\cdot\bar{\bar{P}}\cdot\bar{Z} = \frac{1}{\sqrt{2}} \left\{ X_1(Z_1+Z_3) + X_2(Z_2+Z_4) + X_3(Z_1-Z_3) + X_4(Z_2-Z_4) \right\} \tag{C23}$$

Therefore

$$f(\bar{X}) = J_1\, J_2\, J_3\, J_4 \tag{C24}$$

where

$$J_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dZ_1 \exp \left[-\hbar_1^2 Z_1^2 - i(X_1+X_3)Z_1/\sqrt{2} \right] \tag{C25a}$$

$$J_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dZ_2 \exp \left[-\hbar_1^2 Z_2^2 - i(X_2+X_4)Z_2/\sqrt{2} \right] \tag{C25b}$$

$$J_3 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dZ_3 \exp \left[- i(X_1-X_3)Z_3/\sqrt{2} \right] \tag{C25c}$$

$$J_4 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dZ_4 \exp \left[- i(X_2-X_4)Z_4/\sqrt{2} \right] \tag{C25d}$$

All the above integrals can be readily evaluated and the results are

$$J_1 = \frac{1}{\sqrt{(4\pi)}\, \hbar_1} \exp \left[- \frac{X_1^2}{2\hbar_1^2} \right]$$

$$= \frac{1}{\sqrt{2}} f(X_1) \tag{C26}$$

$$J_2 = \frac{1}{\sqrt{2}} f(X_2) \tag{C27}$$

$$J_3 = \sqrt{2}\, \delta(X_1-X_3) \tag{C28}$$

$$J_4 = \sqrt{2}\, \delta(X_2-X_4) \tag{C29}$$

So finally

$$f(\bar{X}) = f(X_1)\, f(X_2)\, \delta(X_1-X_3)\, \delta(X_2-X_4) \tag{C30}$$

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Nomenclature

C	correlation function
f	normal density function
g	conditional density function
h	cumulant function
h_1	large scale rms height of the surface
h_2	small scale rms height of the surface
$\overline{\overline{K}}$	covariance matrix
ℓ_1	large scale correlation length of the surface
ℓ_2	small scale correlation length of the surface
p, q	slopes along the transmitter and receiver directions
$R_{V,H}$	Fresnel reflection coefficient for vertical, horizontal polarization
γ_{cd}	bistatic scattering cross section
s_1	large scale rms slope of the surface
s_2	small scale rms slope of the surface
μ	slope of the ray emanating from I towards the transmitter
ν	slope of the ray emanating from I towards the receiver
$\xi(x,y)$	random function describing the z-coordinate of the surface
ϕ	azimuthal angle

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